# **Example 7e:** Woven Composite Analysis – Two Step

This example considers the previous plain weave graphite/epoxy composite considered in Example 7d. In this example, however, a two step approach is employed to homogenize the triply periodic RUC in order to arrive at the effective properties of the composite. This two step homogenization procedure has been shown to yield significantly more accurate effective properties for the composite than the one step approach demonstrated in Example 7d (Bednarcyk, 2000).

Considering the plain weave reinforced composite shown in Figure 7.8, the six unique through-thickness subcell groups shown in Figure 7.10 can be identified. Note that reversing the stacking sequence of Group 1 does not result in a unique through-thickness subcell group. In step one of the two step homogenization procedure, the six unique subcell groups are individually homogenized in the through-thickness direction. The result is a set of effective thermo-elastic properties for each of the groups. Note that, since Group 5 consists only of matrix material, its effective properties are that of the matrix. Further, Groups 2, 3, 4, and 6 contain subcells with inclined fibers. Thus, the effective properties of these Groups will be monoclinic.

Once the effective properties of each through-thickness subcell group are determined, an RUC, with only one through-thickness subcell, can be assembled that represents the woven composite. This is shown in Figure 7.11, where the effective properties of the appropriate through-thickness group are used to represent the subcell materials. Then, as step two of the procedure, this two-dimensional RUC is homogenized to determine the effective properties of the woven composites. As will be shown, the results of this two step homogenization procedure can be quite different from those of the corresponding one step homogenization procedure.

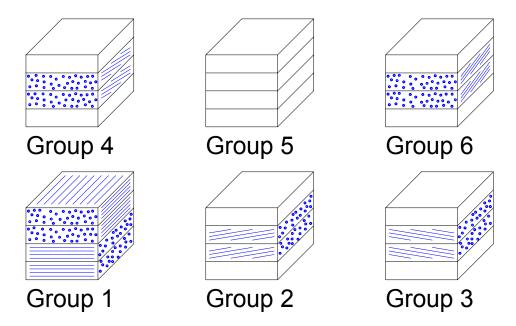
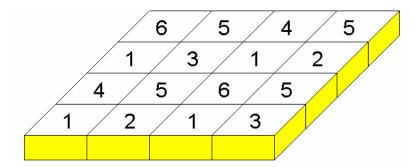


Figure 7.10 Unique through-thickness groups in the plain weave reinforced composite shown in Figure 7.8. These through-thickness subcell groups are homogenized using MAC/GMC 4.0 to determine their effective stiffness matrices in Step 1 of the two step homogenization procedure.



**Figure 7.11** Representation of the plain-weave reinforced composite after though-thickness homogenization in Step 1. Step 2 involves homogenizing this RUC to determine the effective properties of the plain weave reinforced composite.

## MAC/GMC Input Files: example 7e 1.mac, example 7e 2.mac

```
MAC/GMC 4.0 Example 7e 1 - Two step plain weave composite analysis (step 1)
*CONSTITUENTS
 NMATS=7
 M=1 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.,0.,1.
 M=2 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.,1.,0.
 M=3 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.25,1.,0.
 M=4 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=-0.25,1.,0.
 M=5 CMOD=6 MATID=U MATDB=1 &
  EL=3.45E9,3.45E9,0.35,0.35,1.278E9,45.E-6,45.E-6
 M=6 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.25,0.,1.
 M=7 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=-0.25,0.,1.
 MOD=3 ARCHID=99
 NA=4 NB=1 NG=1
 D=0.25,0.25,0.25,0.25
 H=1.
 L=1.
# -- Group 1
  SM=1
  SM=1
  SM=2
  SM=2
# -- Group 2
 SM=5
# SM=3
# SM=3
# SM=5
# -- Group 3
# SM=5
```

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```
# SM=4
# SM=4
# SM=5
# -- Group 4
# SM=5
# SM=7
# SM=7
# SM=5
# -- Group 6
# SM=5
# SM=6
# SM=6
# SM=5
*PRINT
  NPL = -1
*END
MAC/GMC 4.0 Example 7e 2 - Two step plain weave composite analysis (step 2)
*CONSTITUENTS
    NMATS=6
    M=1 CMOD=15 MATID=U MATDB=1 &
      EL=7.827E9, 4.103E9, 4.103E9, 0., 0., 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., & 0., 
                                                                                                                          0., &
                                                                                      4.167E9, 0.,
                                                                                                               2.757E9, 0., &
                                                                                                                                       2.757E9 &
                38.56E-6, 0.3901E-6, 0.3901E-6, 0., 0., 0.
     M=2 CMOD=15 MATID=U MATDB=1 &
       EL=6.677E9, 4.843E9, 3.274E9, 0., 0.,
                                                                                                                                     0.1887E9, &
                                       55.80E9, 3.384E9, 0., 0., 6.969E9, & 6.662E9, 0., 0., -0.006711E9, &
                                                                                       2.625E9, 0.1830E9, 0., &
                                                                                                               1.613E9, 0., &
                                                                                                                                          2.269E9 &
               43.74E-6, 4.043E-6, 39.04E-6, 0., 0., -12.19E-6
    M=3 CMOD=15 MATID=U MATDB=1 &
       EL=6.677E9, 4.843E9, 3.274E9, 0., 0., -0.1887E9, &
                                       55.80E9, 3.384E9, 0., 0., -6.969E9, & 6.662E9, 0., 0., 0.006711E9, &
                                                                                       2.625E9,-0.1830E9, 0., &
                                                                                                               1.613E9, 0., &
                                                                                                                                          2.269E9 &
                43.74E-6, 4.043E-6, 39.04E-6, 0., 0., 12.19E-6
     M=4 CMOD=15 MATID=U MATDB=1 &
       EL=6.677E9, 3.274E9, 4.843E9, 0., -0.1887E9, 0., &
                                       6.662E9, 3.384E9, 0., 0.006711E9, 0., & 55.80E9, 0., -6.969E9, 0., &
                                                                                       2.625E9, 0.,
                                                                                                                                          -0.1830E9, &
                                                                                                              2.269E9, 0., &
                                                                                                                                              1.613E9 &
                43.74E-6, 39.04E-6, 4.043E-6, 0., 12.19E-6, 0.
     M=5 CMOD=6 MATID=U MATDB=1 &
      EL=3.45E9,3.45E9,0.35,0.35,1.278E9,45.E-6,45.E-6
    M=6 CMOD=15 MATID=U MATDB=1
      NTP=2
```

```
TEM=23.,150.
   C11=6.677E9,6.677E9
   C12=3.274E9,3.274E9
  C13=4.843E9,4.843E9
  C14=0.,0.
  C15=0.1887E9, 0.1887E9
  C16=0.,0.
   C22=6.662E9,6.662E9
   C23=3.384E9,3.384E9
   C24=0.,0.
   C25=-0.006711E9,-0.006711E9
   C26=0.,0.
   C33=55.80E9,55.80E9
   C34=0.,0.
   C35=6.969E9,6.969E9
   C36=0.,0.
   C44=2.625E9, 2.625E9
  C45=0.,0.
   C46=0.1830E9, 0.1830E9
   C55=2.269E9,2.269E9
   C56=0.,0.
   C66=1.613E9, 1.613E9
  ALF1=43.74E-6,43.74E-6
  ALF2=39.04E-6,39.04E-6
  ALF3=4.043E-6,4.043E-6
  ALF4=0.,0.
  ALF5=-12.19E-6,-12.19E-6
  ALF6=0.,0.
*RUC
 MOD=3 ARCHID=99
 NA=1 NB=4 NG=4
 D=1.
 H=1.,1.,1.,1.
 L=1.,1.,1.,1.
 SM=1,2,1,3
 SM=4,5,6,5
  SM=1,3,1,2
 SM=6,5,4,5
*PRINT
 NPL=-1
*END
```

## **Annotated Input Data**

### Step 1: Homogenization of the through-thickness subcell groups → example 7e 1.mac

- 1) Flags: None
- 2) Constituent materials (\*CONSTITUENTS) [KM\_2]:

Number of materials: 7 (NMATS=7)

Constitutive models: Arbitrary transversely isotropic (CMOD=9)

Elastic (CMOD=6)

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Materials:	User-defined	(MATID=U)
Material property source:	Read from input file	(MATDB=1)
Material properties:	See Table 7.1	(EL=)
Direction of trans. Isotropy	: Material #1: (0, 0, 1)	(D=0.,0.,1.)
	Material #2: (0, 1, 0)	(D=0.,1.,0.)
	Material #3: (0.25, 1, 0)	(D=0.25, 1., 0.)
	Material #4: (-0.25, 1, 0)	(D=-0.25,1.,0.)
	Material #6: (0.25, 0, 1)	(D=0.25, 0., 1.)
	Material #7: (-0.25, 0, 1)	(D=-0.25,0.,1.)

In this example problem, the constituent materials are the same as those employed in Example 7d.

3) Analysis type (\*RUC)  $\rightarrow$  Repeating Unit Cell Analysis [KM 3]:

Analysis model:	Triply periodic GMC	(MOD=3)
RUC architecture:	User-defined	(ARCHID=99)
No. subcells in x1-dir.:	4	(NA=4)
No. subcells in x2-dir.:	1	(NB=1)
No. subcells in x3-dir.:	1	(NG=1)
Subcell depths:	0.25, 0.25, 0.25, 0.25	(D=0.25,0.25,0.25,0.25)
Subcell height:	1.	(H=1.)
Subcell length:	1.	(L=1.)
Material assignment:	see input file	(SM=)

In this example problem, effective elastic properties must be generated for each of five subcell groups. Recall that Group 5 contains only the matrix material and thus does not need to be homogenized. The appropriate lines in the input file must be commented and uncommented in order to generate all of these effective properties.

- 4) Loading: None
- 5) Damage and Failure: None
- 6) Output:
  - a) Output file print level (\*PRINT) [KM\_6]:

Print level: -1 (effective properties only) (NPL=-1)

- b) x-y plots (**\*XYPLOT**): None
- 7) End of file keyword: (\*END)

## Step 2: Homogenization of the 2D RUC → example\_7e\_2.mac

- 1) Flags: None
- 2) Constituent materials (\*CONSTITUENTS) [KM 2]:

Number of materials:	6	(NMATS=6)
Constitutive models:	Anisotropic elastic	(CMOD=15)
	Elastic	(CMOD=6)

Materials: User-defined (MATID=U)
Material property source: Read from input file (MATDB=1)
Material properties: See input file (EL=...)

In step two of this example problem, the material occupying many of the subcells represents homogenized subcell groups that contain subcells with inclined fibers (see Figure 7.10). These materials are monoclinic and thus require a more general elastic constitutive model than presented this far in this Example Manual. For this reason, a completely anisotropic elastic model (CMOD=15) has been incorporated within MAC/GMC 4.0. This model allows the user to specify all 21 components of the constituent material stiffness matrix, as well as six CTE components. For material #6, the use of this model in conjunction with temperature-dependent material properties is illustrated. For more information on constituent materials, see the MAC/GMC 4.0 Keywords Manual Section 2.

3) Analysis type (\*RUC)  $\rightarrow$  Repeating Unit Cell Analysis [KM 3]:

Analysis model: Triply periodic GMC (MOD=3)User-defined RUC architecture: (ARCHID=99) No. subcells in x1-dir.: 1 (NA=1)No. subcells in x2-dir.: 4 (NB=4)No. subcells in x3-dir.: 4 (NG=4)Subcell depth: 1. (D=1.)1., 1., 1., 1. (H=1.,1.,1.,1.)Subcell heights: Subcell lengths: 1., 1., 1., 1. (L=1.,1.,1.,1.)Material assignment: see input file (SM=...)

The anisotropic materials that represent the homogenized subcell groups are arranged as shown in Figure 7.11 to represent the plain weave reinforced composite. Note that the anisotropic constitutive model can only be used with triply-periodic GMC (i.e., the doubly periodic GMC implementation assumes the subcell materials to be at most orthotropic in the RUC coordinate system). Thus, even though the RUC in this example problem is two-dimensional, triply periodic GMC (with one  $x_1$ -direction subcell) has been employed. For more information on the MAC/GMC 4.0 RUC capabilities, see the MAC/GMC 4.0 Keywords Manual Section 3.

- 4) Loading: None
- 5) Damage and Failure: None
- 6) Output:
  - a) Output file print level (\*PRINT) [KM 6]:

Print level: -1 (effective properties only) (NPL=-1)

- b) x-y plots (\*XYPLOT): None
- 7) End of file keyword: (\*END)

## Results

Results for this example problem take the form of the effective thermo-elastic properties of the subcell groups (in step 1) and of the plain weave reinforced graphite/epoxy composite (step 2). The effective stiffness matrix, elastic moduli, and CTEs of each through-thickness subcell group (determined in step 1) are given below. Because Group 1 contains no subcells with inclined fibers, the group's effective thermo-elastic properties are orthotropic. The remaining subcell groups (with the exception of Group 5, which contains only the matrix material and thus does not need to be homogenized in step 1) do contain subcells with inclined fibers. The effective elastic behavior of these homogenized subcell groups is monoclinic (i.e., 13 independent constants, see Jones (1975)). The non-zero  $C_{16}$  term for Group 2 indicates that, for example, a non-zero shear stress  $\sigma_{12}$  component would arise were a normal strain component  $\varepsilon_{11}$  applied. In addition, each monoclinic subcell group has a non-zero shear CTE. This indicates that, were a temperature change applied to the subcell group, a shear strain component would arise. The effective stiffness matrix of each group, as well as the effective CTEs, are employed as constituent material properties in step 2 of the homogenization procedure.

### **GROUP 1**

```
CG - Effective/Macro Stiffness Matrix
   NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS
   0.7827E+10 0.4103E+10 0.4103E+10 0.0000E+00 0.0000E+00 0.0000E+00
   0.4103E+10 0.1324E+12 0.4519E+10 0.0000E+00 0.0000E+00 0.0000E+00
   0.4103E+10 0.4519E+10
0.0000E+00 0.0000E+00
                             0.1324E+12 0.0000E+00 0.0000E+00
0.0000E+00 0.4167E+10 0.0000E+00
                                                                     0.0000E+00
                                                                     0.0000E+00
   0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.2757E+10 0.0000E+00
   0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.2757E+10
Effective Engineering Moduli
   Use with caution when global stiffness matrix is anisotropic
    E11S= 0.7581E+10
    N12S= 0.0300
    E22S= 0.1302E+12
    N23S = 0.0182
    E33S= 0.1302E+12
    G23S= 0.4167E+10
     G13S= 0.2757E+10
    G12S= 0.2757E+10
Effective Thermal Expansion Coefficients
  NOTE: Shear CTEs are "engineering" shear CTEs
   0.3856E-04 0.3901E-06 0.3901E-06
   0.0000E+00 0.0000E+00 0.0000E+00
GROUP 2
CG - Effective/Macro Stiffness Matrix
   NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS
   0.6677E+10 0.4843E+10 0.3274E+10 0.0000E+00 0.0000E+00 0.1887E+09
   0.4843E+10 0.5580E+11 0.3384E+10 0.0000E+00 0.0000E+00 0.6969E+10
   0.3274E+10 0.3384E+10 0.6662E+10 0.0000E+00 0.0000E+00 -0.6711E+07 0.0000E+00 0.0000E+00 0.0000E+00 0.2625E+10 0.1830E+09 0.0000E+00
   0.0000E+00 0.0000E+00 0.0000E+00 0.1830E+09 0.1613E+10 0.0000E+00
   0.3775E+09 0.5575E+10 -0.5369E+07 0.0000E+00 0.0000E+00 0.2269E+10
```

```
Effective Engineering Moduli
    Use with caution when global stiffness matrix is anisotropic
     E11S= 0.4889E+10
     N12S= 0.0733
     E22S= 0.3586E+11
     N23S= 0.3140
     E33S= 0.4994E+10
     G23S = 0.2604E + 10
     G13S= 0.1600E+10
     G12S= 0.1552E+10
 Effective Thermal Expansion Coefficients
   NOTE: Shear CTEs are "engineering" shear CTEs
    0.4374E-04 0.4043E-05 0.3904E-04
    0.0000E+00 0.0000E+00 -0.1219E-04
GROUP 3
CG - Effective/Macro Stiffness Matrix
    NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS

    0.6677E+10
    0.4843E+10
    0.3274E+10
    0.0000E+00
    0.0000E+00
    -0.1887E+09

    0.4843E+10
    0.5580E+11
    0.3384E+10
    0.0000E+00
    0.0000E+00
    -0.6969E+10

    0.3274E+10
    0.3384E+10
    0.6662E+10
    0.0000E+00
    0.0000E+00
    0.6711E+07

   Effective Engineering Moduli
    Use with caution when global stiffness matrix is anisotropic
     E11S= 0.4889E+10
    N12S = 0.0733
    E22S= 0.3586E+11
     N23S = 0.3140
     E33S= 0.4994E+10
     G23S= 0.2604E+10
     G13S= 0.1600E+10
     G12S= 0.1552E+10
Effective Thermal Expansion Coefficients
   NOTE: Shear CTEs are "engineering" shear CTEs
    0.4374E-04 0.4043E-05 0.3904E-04
    0.0000E+00 0.0000E+00 0.1219E-04
GROUP 4
 CG - Effective/Macro Stiffness Matrix
    NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS
                 0.6677E+10
                                                                         0.0000E+00
   0.3274E+10 0.6662E+10 0.3384E+10 0.0000E+00 0.6711E+07 0.0000E+00
   0.4843E+10 0.3384E+10 0.5580E+11 0.0000E+00 -0.6969E+10 0.0000E+00
   0.0000E+00 0.0000E+00 0.0000E+00 0.2625E+10 0.0000E+00 -0.1830E+09 -0.3775E+09 0.5369E+07 -0.5575E+10 0.0000E+00 0.2269E+10 0.0000E+00
```

0.0000E+00 0.0000E+00 0.0000E+00 -0.1830E+09 0.0000E+00 0.1613E+10

```
Effective Engineering Moduli
    Use with caution when global stiffness matrix is anisotropic
     E11S= 0.4889E+10
     N12S= 0.4541
     E22S= 0.4994E+10
     N23S = 0.0343
     E33S= 0.3586E+11
     G23S = 0.2604E + 10
     G13S= 0.1552E+10
     G12S= 0.1600E+10
 Effective Thermal Expansion Coefficients
   NOTE: Shear CTEs are "engineering" shear CTEs
    0.4374E-04 0.3904E-04 0.4043E-05
    0.0000E+00 0.1219E-04 0.0000E+00
GROUP 6
 CG - Effective/Macro Stiffness Matrix
    NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS

    0.6677E+10
    0.3274E+10
    0.4843E+10
    0.0000E+00
    0.1887E+09
    0.0000E+00

    0.3274E+10
    0.6662E+10
    0.3384E+10
    0.0000E+00
    -0.6711E+07
    0.0000E+00

    0.4843E+10
    0.3384E+10
    0.0000E+00
    0.6969E+10
    0.0000E+00

    0.0000E+00 0.0000E+00 0.0000E+00 0.2625E+10 0.0000E+00 0.1830E+09
    0.3775E+09 -0.5369E+07 0.5575E+10 0.0000E+00 0.2269E+10 0.0000E+00 0.0000E+00 0.0000E+00 0.1830E+09 0.0000E+00 0.1613E+10
Effective Engineering Moduli
    Use with caution when global stiffness matrix is anisotropic
     E11S= 0.4889E+10
     N12S= 0.4541
     E22S= 0.4994E+10
     N23S = 0.0343
     E33S= 0.3586E+11
     G23S= 0.2604E+10
     G13S= 0.1552E+10
     G12S= 0.1600E+10
Effective Thermal Expansion Coefficients
   NOTE: Shear CTEs are "engineering" shear CTEs
    0.4374E-04 0.3904E-04 0.4043E-05
    0.0000E+00 -0.1219E-04 0.0000E+00
```

## STEP 2 RESULTS (Effective Properties of the Woven Composite)

The results of step 2 in the two step homogenization procedure for the plain weave reinforced graphite/epoxy composite are the predicted effective thermo-elastic properties of the woven composite. As in Example 7d, the effective stiffness matrix results show that, even through Groups 2, 3, 4, and 6 are monoclinic (in the global coordinates of the woven composite), the woven composite is orthotropic. This is because for every subcell that contains inclined fibers, there is a subcell containing fibers with the incline reversed (see Figure 7.9). The anisotropic terms thus add to zero during the GMC triply periodic homogenization procedure. The same is true of the "shear" CTE terms.

Table 7.2 provides a comparison of the MAC/GMC 4.0 prediction for the in-plane (i.e., plane of the woven reinforcement) thermo-elastic properties of the woven composite. The effect of utilizing the 2 step approach rather than the 1 step approach is stunning. The predicted in plane elastic modulus is 3.2 times higher, the predicted in-plane Poisson ratio is 2.3 times lower, and the predicted in-plane CTE is 2.9 times lower. The effect on the predicted in-plane shear modulus is less significant; it is only 7.5% higher when using the 2 step approach. The reason for this dramatic effect on the effective properties is the fact that the triply periodic RUC that represents the woven composite (see Figure 7.8 and Figure 7.9) has no continuous fibers spanning the x<sub>2</sub>- and x<sub>3</sub>-directions. Following any row or column of subcells in the x<sub>2</sub>and  $x_3$ -directions (at a constant  $x_1$  position), a subcell containing either the pure matrix material or transversely oriented fibers will be encountered. These subcells serve as weak links and cause the 1 step homogenization approach to predict a low effective in-plane stiffness (along with a high in-plane Poisson ratio and CTE). In the 2 step homogenization procedure, the woven composite properties are homogenized, or "smeared", in the through-thickness  $(x_1)$  direction in step 1. This, in effect, removes the extreme weak links from the RUC. In Figure 7.11, two rows of subcells in the  $x_2$ -direction exists that contain only homogenized Groups 1, 2, and 3, while in the x<sub>3</sub>-direction, two columns of subcells exist that contain only homogenized Groups 1, 4, and 6 (the remaining two rows or columns in the two directions do contain extreme weak link subcells). Examining Figure 7.10, it is clear that, while Groups 2, 3, 4, and 6 are less stiff than Group 1, they still contain subcells with fibers mainly aligned in the appropriate inplane direction. This fact can also be seen in the effective properties of the Groups given above. Therefore, the RUC depicted in Figure 7.11 has two rows or columns of subcells in each of the in-plane directions that do not contain extreme weak link subcells. The result is the stiffer predicted composite properties obtained in the 2 step approach. Note that the large constituent property mismatch between the graphite fiber and epoxy matrix in the present example magnifies the discrepancy between the 1 step and 2 step approaches. A significant improvement in the 1 step procedure will result from the use of HFGMC rather than GMC to model the woven composite. However, in order to model the architecture of the woven reinforcement, triply periodic HFGMC is required. This theory is currently under development.

Finally, Bednarcyk (2000) showed that the 2 step MAC/GMC homogenization procedure provides much more accurate predictions of woven composite properties than does the 1 step approach. The two step approach is somewhat cumbersome compared to most of the other features within MAC/GMC 4.0. Future versions of MAC/GMC will automate the woven composite analysis procedure in order to relieve this burden from the user.

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CG - Effective/Macro Stiffness Matrix
   NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS
   0.6669E+10 0.3662E+10 0.3662E+10
                                          0.0000E+00 0.5215E-07
                                                                    0.2980E-07
   0.3662E+10 0.3022E+11 0.3562E+10 0.0000E+00 0.2068E-24 0.0000E+00
  0.3662E+10 0.3562E+10 0.3022E+11 0.0000E+00 0.5679E-08 0.0000E+00 0.0000E+00 0.0000E+00 0.2234E+10 0.0000E+00 0.0000E+00
  -0.4657E-08 0.0000E+00 0.3950E-09 0.0000E+00 0.1626E+10 0.0000E+00
  -0.1397E-08 0.4740E-08 0.3725E-08 0.0000E+00 0.0000E+00 0.1626E+10
Effective Engineering Moduli
   Use with caution when global stiffness matrix is anisotropic
   E11S= 0.5875E+10
   N12S= 0.1084
    E22S= 0.2813E+11
   N23S= 0.0550
   E33S= 0.2813E+11
   G23S = 0.2234E + 10
   G13S= 0.1626E+10
   G12S= 0.1626E+10
```

## MAC/GMC 4.0 Example Problem Manual

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Effective Thermal Expansion Coefficients

NOTE: Shear CTEs are "engineering" shear CTEs

0.5595E-04 0.7048E-05 0.7048E-05
0.0000E+00 -0.1206E-21 0.1138E-22
```

**Table 7.2** Predicted effective in-plane thermo-elastic properties for 32.5% plain weave reinforced graphite/epoxy composite using the MAC/GMC 4.0 1 step and 2 step homogenization procedures.

	E (GPa)	ν	G (GPa)	$\alpha (\times 10^{-6} / ^{\circ}\text{C})$
1 Step Approach	8.703	0.1245	2.079	20.45
2 Step Approach	28.13	0.0550	2.234	7.048